Deviations from off-diagonal long-range order in one-dimensional quantum systems

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A quantum system exhibits off-diagonal long-range order (ODLRO)[1] when the largest eigenvalue λ_0 of the one-body-density matrix scales as $\lambda_0 \sim N$, where N is the total number of particles.

Putting $\lambda_0 \sim N^{\mathcal{C}}$ to define the scaling exponent \mathcal{C} , when $0 < \mathcal{C} < 1$, then \mathcal{C} can be used to quantify deviations from ODLRO.

We study the exponent C in a variety of one-dimensional bosonic and anyonic quantum systems at T = 0.

For the one-dimensional Lieb-Liniger Bose gas[2] we find, using bosonization technique, that for small interactions \mathcal{C} is close to 1, implying a mesoscopic condensation, i.e. a value of the zero temperature "condensate" fraction λ_0/N appreciable different from zero at finite values of the number of particles (as the ones in experiments with 1D ultracold atoms). These results are confirmed by numerical method. using known asymptotic expressions for the one-body density matrix, $\rho(x,x')$, of the system and finding a suitable interpolation formula for the evaluation of ρ at any distances, interaction and number of particles. See Fig. 1 for the comparison. One-dimensional anyons[3] provide the possibility to fully interpolate between $\mathcal{C} = 1$ and 0. The behaviour of the power \mathcal{C} for these systems is found to be non-monotonic both with respect to the coupling constant and the statistical parameter, as can be seen from Fig. 2.

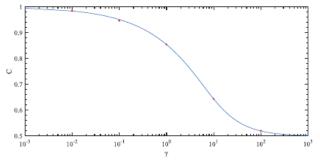


Figure 1: C vs interaction parameter γ for a Lieb-Liniger Bose gas: bosonization prediction (blue line) and numerical results (red dots).

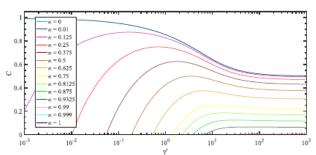


Figure 2: C vs interaction parameter γ' for a Lieb-Liniger anyon gas for different values of the statistical parameter κ .

Keywords: OFF-DIAGONAL LONG-RANGE ORDER, ONE-DIMENSION, LIEB-LINIGER, ANYONS

References

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